Formulas for Mathematics – continued level 2 and formulas for Mathematics 4

Algebra

Rules

$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$(a+b)(a-b) = a^2 - b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$
$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Quadratic equations

$$x^2 + px + q = 0 \qquad x = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

$$ax^2 + bx + c = 0 \qquad x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Arithmetic

Prefixes

T	G	M	k	h	d	c	m	μ	n	p
tera	giga	mega	kilo	hecto	deci	centi	milli	micro	nano	pico
10^{12}	10^{9}	10^{6}	10^{3}	10^{2}	10^{-1}	10^{-2}	10^{-3}	10^{-6}	10^{-9}	10^{-12}

Powers

$$a^x a^y = a^{x+y} \qquad \frac{a^x}{a^y} = a^{x-y} \qquad (a^x)^y = a^{xy} \qquad a^{-x} = \frac{1}{a^x}$$

$$a^x b^x = (ab)^x \qquad \frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x \qquad a^{\frac{1}{n}} = \sqrt[n]{a} \qquad a^0 = 1$$

Geometric series

$$a + ak + ak^2 + \ ...\ + ak^{n-1} = \frac{a(k^n - 1)}{k - 1} \quad \text{where } k \neq 1$$

Logarithms

$$y = 10^x \Leftrightarrow x = \lg y \qquad y = \mathrm{e}^x \Leftrightarrow x = \ln y$$

$$\lg x + \lg y = \lg xy \qquad \lg x - \lg y = \lg \frac{x}{y} \qquad \lg x^p = p \cdot \lg x$$

Absolute value

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Functions and relations

Linear function

$y = kx + m$ $\qquad k = \dfrac{y_2 - y_1}{x_2 - x_1}$

$k_1 \cdot k_2 = -1$, condition for perpendicular lines

$ax + by + c = 0$, where a and b are not both zero

Quadratic functions

$y = ax^2 + bx + c \qquad a \neq 0$

Power functions

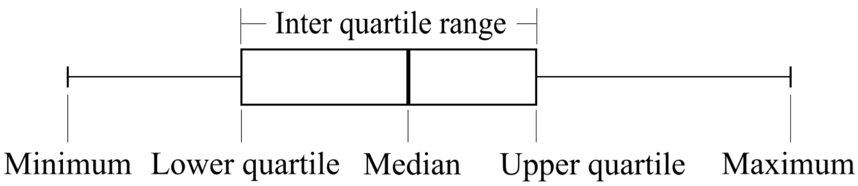
$y = C \cdot x^a$

Exponential functions

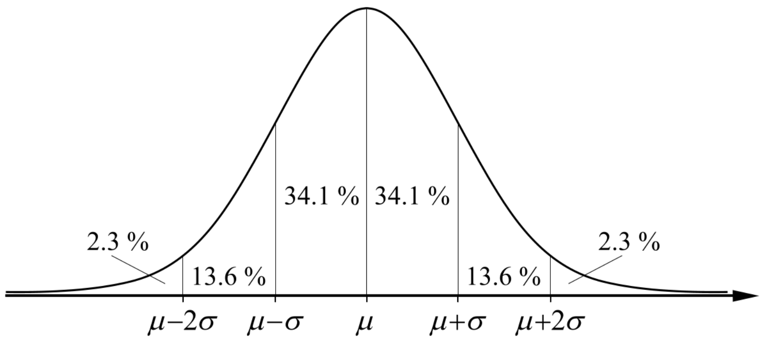
$y = C \cdot a^x \qquad a > 0$ and $a \neq 1$

Statistics and probability

Box plot



Normal distribution



Density function of the normal distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Differential and integral calculus

Definition of the derivative

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Derivatives

Function	Derivative
x^n where n is a real number	nx^{n-1}
$\frac{1}{x}$	$-\frac{1}{x^2}$
$\ln x \quad (x > 0)$	$\frac{1}{x}$
$a^x \quad (a > 0)$	$a^x \ln a$
e^x	e^x
e^{kx}	$k \cdot \mathrm{e}^{kx}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$1 + \tan^2 x = \frac{1}{\cos^2 x}$
$k \cdot f(x)$	$k \cdot f'(x)$
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x) \cdot g(x)$	$f'(x) \cdot g(x) + f(x) \cdot g'(x)$
$\frac{f(x)}{g(x)} \quad (g(x) \neq 0)$	$\frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$

Chain rule

If $y = f(z)$ and $z = g(x)$ are two differentiable functions then it holds for $y = f(g(x))$ that

$$y' = f'(g(x)) \cdot g'(x) \text{ or } \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}z} \cdot \frac{\mathrm{d}z}{\mathrm{d}x}$$

Fundamental theorem of calculus $\int_a^b f(x)\,\mathrm{d}x = [F(x)]_a^b = F(b) - F(a)$ where $F'(x) = f(x)$

Antiderivatives

Function	Antiderivatives
k	$kx + C$
$x^n \quad (n \neq -1)$	$\frac{x^{n+1}}{n+1} + C$
$\frac{1}{x}$	$\ln x + C \quad (x > 0)$
$a^x \quad (a > 0,\ a \neq 1)$	$\frac{a^x}{\ln a} + C$
e^x	$\mathrm{e}^x + C$
e^{kx}	$\frac{\mathrm{e}^{kx}}{k} + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$

Volume of solids of revolution

$V = \pi \cdot \int_a^b y^2\,\mathrm{d}x$ rotation around the x-axis

$V = \pi \cdot \int_a^b x^2\,\mathrm{d}y$ rotation around the y-axis

Complex numbers

Representation

$z = a + b\mathrm{i}$ Rectangular form

$z = r(\cos v + \mathrm{i}\sin v)$ Polar form

$z = r\mathrm{e}^{\mathrm{i}v}$ Exponential form

Argument $\arg z = v \qquad \tan v = \frac{b}{a}$

Absolute value $|z| = r = \sqrt{a^2 + b^2}$

Conjugate $\bar{z} = a - b\mathrm{i}$

Rules

$z_1 \cdot z_2 = r_1 \cdot r_2(\cos(v_1 + v_2) + \mathrm{i}\sin(v_1 + v_2))$

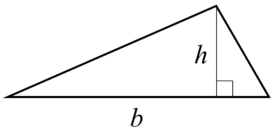
$\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(v_1 - v_2) + \mathrm{i}\sin(v_1 - v_2))$

de Moivre's formula $z^n = (r(\cos v + \mathrm{i}\sin v))^n = r^n(\cos nv + \mathrm{i}\sin nv)$

Geometry

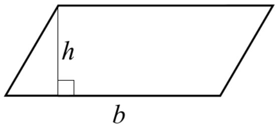
Triangle

$$A = \frac{bh}{2}$$



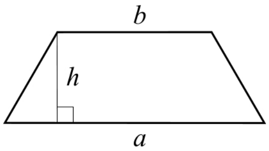
Parallelogram

$$A = bh$$



Trapezium

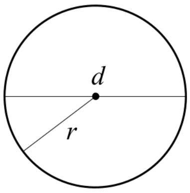
$$A = \frac{h(a+b)}{2}$$



Circle

$$A = \pi r^2 = \frac{\pi d^2}{4}$$

$$O = 2\pi r = \pi d$$



Circle sector

v is measured in degrees

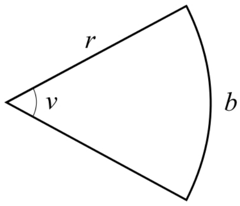
$$b = \frac{v}{360^\circ} \cdot 2\pi r$$

$$A = \frac{v}{360^\circ} \cdot \pi r^2 = \frac{br}{2}$$

v is measured in radians

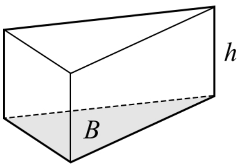
$$b = vr$$

$$A = \frac{vr^2}{2} = \frac{br}{2}$$



Prism

$$V = Bh$$

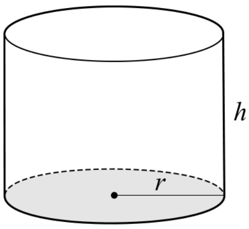


Cylinder

$$V = \pi r^2 h$$

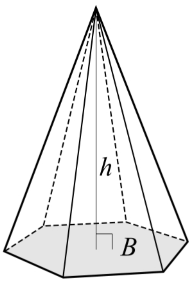
Lateral surface area

$$A = 2\pi rh$$



Pyramid

$$V = \frac{Bh}{3}$$

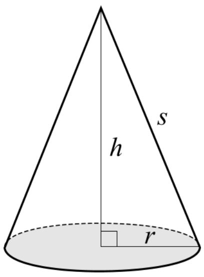


Cone

$$V = \frac{\pi r^2 h}{3}$$

Lateral surface area

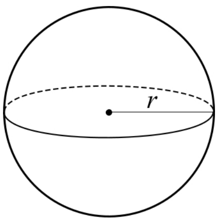
$$A = \pi rs$$



Sphere

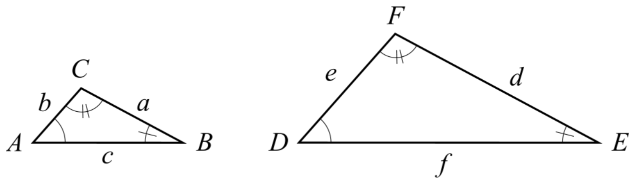
$$V = \frac{4\pi r^3}{3}$$

$$A = 4\pi r^2$$



Similarity

The triangles ABC and DEF are similar if $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$

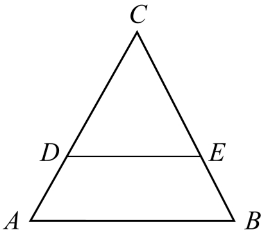


Scale Area scale factor = (Length scale factor)^2 Volume scale factor = (Length scale factor)^3

Triangle with a transversal line

$\frac{DE}{AB} = \frac{CD}{AC} = \frac{CE}{BC}$ and

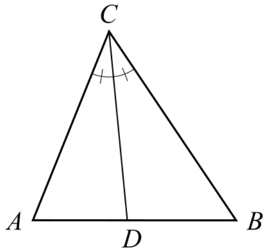
$\frac{CD}{AD} = \frac{CE}{BE}$



DE is parallel to AB

Angle bisector theorem

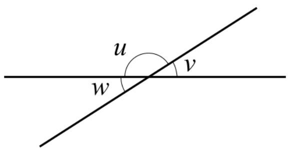
$\frac{AD}{BD} = \frac{AC}{BC}$



Angles

$u + v = 180°$ Supplementary angles

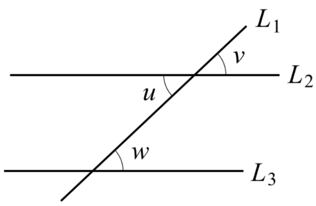
$w = v$ Vertical angles



L_1 intersects two parallel lines L_2 and L_3

$v = w$ Corresponding angles

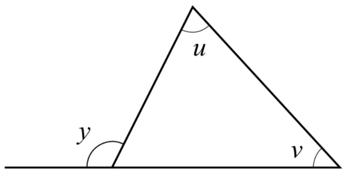
$u = w$ Alternate angles



The sum S of all angles of a n-polygon: $S = (n-2) \cdot 180°$

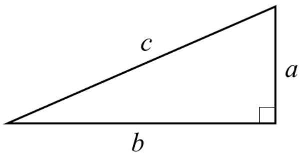
Exterior angle theorem

$y = u + v$



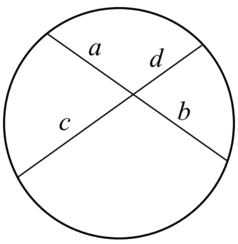
Pythagoras' theorem

$a^2 + b^2 = c^2$



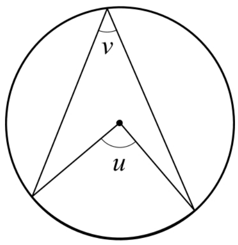
Chord theorem

$ab = cd$



Angles subtended by the same arc

$u = 2v$



Distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

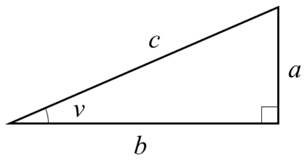
Midpoint formula

$$x_m = \frac{x_1 + x_2}{2} \text{ and } y_m = \frac{y_1 + y_2}{2}$$

Trigonometry

Definitions

Right-angled triangle

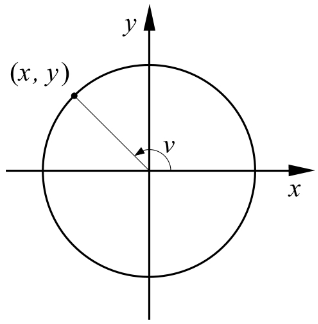


$$\sin v = \frac{a}{c}$$

$$\cos v = \frac{b}{c}$$

$$\tan v = \frac{a}{b}$$

Unit circle



$$\sin v = y$$

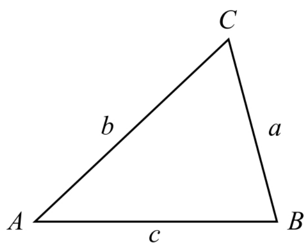
$$\cos v = x$$

$$\tan v = \frac{y}{x}$$

Law of sines $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Law of cosines $a^2 = b^2 + c^2 - 2bc\cos A$

Area formula $T = \frac{ab\sin C}{2}$



Trigonometric formulas

$$\sin^2 v + \cos^2 v = 1$$

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u-v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u-v) = \cos u \cos v + \sin u \sin v$$

$$\sin 2v = 2\sin v \cos v$$

$$\cos 2v = \begin{cases} \cos^2 v - \sin^2 v & (1) \\ 2\cos^2 v - 1 & (2) \\ 1 - 2\sin^2 v & (3) \end{cases}$$

$$a\sin x + b\cos x = c\sin(x+v) \quad \text{where} \quad c = \sqrt{a^2+b^2} \quad \text{and} \quad \tan v = \frac{b}{a}$$

Values of trigonometric functions

Angle v (degrees) (radians)	$0°$ 0	$30°$ $\frac{\pi}{6}$	$45°$ $\frac{\pi}{4}$	$60°$ $\frac{\pi}{3}$	$90°$ $\frac{\pi}{2}$	$120°$ $\frac{2\pi}{3}$	$135°$ $\frac{3\pi}{4}$	$150°$ $\frac{5\pi}{6}$	$180°$ π
$\sin v$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\cos v$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
$\tan v$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not def.	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0